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ABSTRACT

Fourteen research reports related to mathematics education are abstracted and analyzed. Two of the reports deal with teacher education, two with problem solving, three with basic operations, and one each with learning disabled students, rational numbers, proportional reasoning, counting, teacher effectiveness, group cooperation on mathematical tasks, and verbal problem solving. Research related to mathematics education which was reported in RIE and CIJE between October and December 1979 is also listed. (MK)

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Spring 1980

Bright, George W. IDENTIFICATION OF EMBEDDED FIGURES BY PROSPECTIVE ELEMENTARY SCHOOL TEACHERS. School Science and Mathematics 79: 322-327; April 1979.

Abstract and comments prepared for I.M.E. by ARTHUR F. COXFORD, University of Michigan.

1. Purpose

The goal was to determine whether prospective elementary school teachers could identify certain geometry figures when the figure was embedded in a more inclusive figure. In particular, were overlapping figures identified as easily as non-overlapping figures; were embedded figures identified in random order; and were embedded triangles and quadrilaterals equally easy to identify?

2. Rationale

In previous work Bright found that the performances of young children ($\bar{X} = 8.1$ years) in identifying overlapping and and non-overlapping figures were not the same. The investigation suggested that one explanation for the performance differences might have been that elementary school teachers could not teach the skill because they could not identify overlapping figures themselves.

3. Research Design and Procedures

A set of four quadrilateral drawings and a set of four triangle drawings which included overlapping and non-overlapping figures were prepared. The relevant vertices were lettered. One set of drawings was randomly distributed to each of 145 prospective elementary school teachers in university methods and mathematical content classes. The subjects were instructed to isolate all triangles or quadrilaterals in each figure and to copy each figure so isolated in numbered spaces provided on the page with the drawing. Thirty minutes were allowed. The data were analyzed to determine the numbers of overlapping and non-overlapping figures identified and the order in which they were found.

4. Findings

For the two most complex triangle figures, 95 percent and 51 percent of the non-overlapping and overlapping triangles were identified. Similarly, 92 percent and 49 percent of the non-overlapping and overlapping quadrilaterals were identified. The differences in each case were significant ($p < .001$).

With regard to the order of identifying the figures, 59 percent identified the four non-overlapping figures first in a comparable pair of triangles and quadrilateral figures, and 63 percent identified the non-overlapping figures first in a second set of two comparable figures. Finally, the comparisons of identification rates for triangles and quadrilaterals showed that more triangles than quadrilaterals were identified ($p < .01$).

5. Interpretations

The author concluded that the three specific questions were answered negatively. Non-overlapping figures were more easily identified and generally were identified first, and triangles were more easily identified than quadrilaterals.

The author suggested that the results led to the following educational implications:

a) Since prospective elementary teachers experience difficulty identifying overlapping figures, they cannot be expected successfully to guide students in similar activities.

b) Since quadrilaterals are less well identified than triangles and both are important simple figures, attention needs to be given to learning activities which involve general as well as special quadrilaterals.

c) More attention may need to be directed toward developing problem-solving performance of prospective elementary school teachers, since only about 50 percent of the subjects correctly completed the four most complex figures.

Abstractor's Comments

The rates of identifying overlapping and non-overlapping triangles and quadrilaterals are interesting in themselves. As geometry teachers are well aware, picking out simple figures from more complex figures is not easy for many students. It is not unexpected, then, that the data reported here confirm this informal observation.

The author would have been on more solid ground in his report if he had discussed the order in which the students recorded the figures they found. Data should be generated to decide the issue of the correlation between the record and the actual order in which the figures were recognized.

A second criticism pertains to the broad implications the author drew from the data gathered. Certainly it is not clear that the moderate performance in identifying overlapping triangles and quadrilaterals is due to inability to identify such figures. An alternate hypothesis could be that which figures were identified was a function of the way the students interpreted the instructions which were given. Before claiming inability or difficulty, further data need to be gathered to dispose of alternate hypotheses.

The final two implications are, in the reviewer's opinion, reasonable but seem to follow only indirectly from the data. Again alternate hypotheses are available which need to be investigated.

The study has interesting data which should be used to generate other conjectures about identification of overlapping figures. The data are not sufficient to draw firm conclusions. Implications for classroom practice and teacher education should be withheld until additional data are gathered.

Bright, George W.; Harvey, John G.; and Wheeler, Margariete Montague. USING GAMES TO RETRAIN SKILLS WITH BASIC MULTIPLICATION FACTS. Journal for Research in Mathematics Education 10: 103-110; March 1979.

Abstract and comments prepared for I.M.E. by ROBERT ASHLOCK and DOUGLAS EDGE, University of Maryland.

1. Purpose

The purpose of this study was "to investigate the use of games as an instructional procedure in the retraining skills with basic multiplication facts."

2. Rationale

A summary of research on the cognitive effects of games in mathematics learning shows "that the research in this area is sparse, that the findings are fragmentary and inconclusive, and that there have been no systematic attempts to discover the effects of games." Yet elementary school teachers assume that games assist learning and use them regularly, as evidenced by a cited study.

3. Research Design and Procedures

Two similar studies were conducted during the first ten days of the school year, one in 1976 and one in 1977. Regarding subjects, "The 1976 sample was 14 intact classes (2 fourth-grade, 4 fifth-grade, 2 fifth/sixth-grade, and 6 sixth-grade) from three elementary schools. The 1977 sample was 10 intact classes (3 fifth-grade, 4 fifth/sixth-grade, and 3 sixth-grade) from two elementary schools." The subjects were from white, middle-socioeconomic families.

Both studies used a teams-games-tournament model as a scheme for classroom organization for instructional games. Two games from Developing Mathematical Processes (DMP) were used, MULTIG and DIVTIG. Cooperating teachers received training for their participation in the study, and "agreed to refrain from teaching basic multiplication facts for the duration of the study."

In 1976, MULTIG was assigned to the fourth-grade classes, MULTIG and DIVTIG were randomly assigned (by grade level) to half of the remaining classes, and DIVTIG only to the other classes. "On the first day, a 20-item, 15-minute power pretest of basic multiplication facts was given

to a random selection of one-half of the students." Of the 20 items, 10 were selected from the 36 multiplication facts used in MULTIG; the other 10 were selected from the remaining 64 facts. A placebo addition test was given to the other students. On the second day, teachers demonstrated the appropriate game and explained the tournament and its structure. "During days 3-9, students played the game in the tournament structure and maintained records." On the final day, all students were given a posttest equivalent to the multiplication pretest.

In 1977, MULTIG and DIVTIG were randomly assigned by grade level to half of the classes. On the first day, all students were given a 5-minute, 100-item speed test followed by a power test. For the next eight days, the sequence of events paralleled those of 1976. On the tenth day, all students were administered a 5-minute, 100-item speed test followed by a power test. The power tests were equivalent to the 15-minute, 20-item posttest given in 1976.

In order to test the primary research question for both studies ("Can games be used to retrain skills with basic facts?"), Wilcoxon matched-pairs, signed-rank test statistics (Wilcoxon-T) were computed "for the subscore of game-specific items on the power test, for the total score on the power test, and, in the 1977 study, for the speed test score." To test the first two secondary research questions ("Does pretesting on the skills alter the posttest performance of students?"), "the researchers ran an analysis of variance on the data of each class, with the two levels of treatment defined as pretest and no pretest." Also, "F statistics were computed for each class, one for the game-specific items on the power posttest, and one for the total power posttest score." To test the other secondary research question ("Are retraining effects altered by the use of different drill and practice games?"), an ANOVA was run "on the 1976 means of all but the fourth-grade classes."

4. Findings

For each of the two years studied, the authors present pretest and posttest means and standard deviations. For 1976, the pretest game-specific items score (10 items only) and total-items score (all 20 items) are based on only half of each class, as the other half of the class was given the placebo addition test. The posttest means and the standard

deviations are based on the full class size. Examination of the 1976 results showed, for the game-specific items, some increase in the means for 11 of the 14 classes, no change for one class, and some moderate decrease for two classes. For the total item scores, 12 of the 14 classes showed some increase in the mean scores; the other two showed decreases. These mean differences between the pretest and posttest scores (using the Wilcoxon-T) were, however, significant at the .025 level in favor of the posttest scores.

For 1977, the pretests and posttests were given to the full classes. Examination of these results showed that on both the game-specific items score and the total items score, there were no decreases from pretest to posttest mean scores. Again the differences were moderately small, yet using the Wilcoxon-T they were significant at the .001 level. Pre- and post-speed test scores were also given. Differences were significant, again at the .001 level, but in this case the differences appear much larger. Fifth-grade classes showed an average increase of 129 percent; combined fifth-sixth-grade classes showed an average increase of 23 percent; and the sixth-grade classes showed an average increase of 27 percent. The overall average percentage increase was 56 percent.

Using the 1976 data, none of the 28 F-statistics calculated to determine the effect of pretesting game-specific items and total items was significant at the .05 level. The F-statistics obtained to compare performance between the two-game treatment (MULTIG and DIVTIG) and the one-game treatment (DIVTIG only) also were not significant at the .05 level.

5. Interpretations

The authors indicated, primarily on the basis of the "substantial increases in the speed test scores", that it was reasonable to conclude that the game treatment is an effective way to help students retrain their skills with the basic multiplication facts.

Abstractors' Comments

The authors have presented a very clear rationale for conducting these studies. Opinions about the use of games, like many other beliefs in education, are generally accepted in practice but have not been ade-

quately researched. The primary research question and the secondary research questions do follow directly from their rationale.

The authors stated that their interpretations must be considered in the context of several limitations. Two of these limitations, owing to their importance, require special comment. It was acknowledged that the study was conducted during the first ten instructional days of the school year. Different children return from their summer holidays with different expectations. Some children are eager to return to school; others anticipate a return to "drudgery". This latter group especially might be very excited about the games-tournament approach. The very reason that many teachers choose to use games is for the motivation and competition they provide. These may be factors that would not be as significant at other, less atypical, times of the school year.

The second limitation that should be noted is that no attempt was made to compare the effects of games with any other treatment. The total absence of a control group is somewhat surprising. Simply returning to school and being required to do "school-thinking", which presumably makes some use of numbers, might improve one's facility with all previously trained basic facts. Even if this increase is not as large as that recorded by a games treatment, it might be large enough to mitigate against finding the games treatment significant at the .05 level.

The readability of the article could have been improved somewhat by more careful attention to providing brief, explanation-oriented phrases or sentences. Several examples are illustrated:

a) The authors wrote under the procedures section that "a random selection of one-half of the students" were given the 15-minute power pretest. Clarification of whether this is one-half of total students or per class is only available by reading the fine print under one of the tables, "the pretest was given randomly to half of each class", or by interpreting a sentence listed under the results section, "... the researchers ran an analysis of variance on the data of each class, with two levels of treatment defined as pretest and no pretest".

b) During a description of the treatment, the authors indicated that "... During days 3-9, students played the games in the tournament structure and maintained records". No mention of

the amount of time, per day, was provided. Casually, only to support an argument, was the "15-minutes a day" mentioned in one of the final paragraphs of the articles.

c) Very careful reading (and likely rereading) is necessary to understand:

> "In 1976, MULTIG was assigned to both fourth-grade classes. MULTIG and DIVTIG were randomly assigned, by grade level, to half of the remaining classes. DIVTIG was used with the other classes."

It might have been helpful to indicate "... the two-game treatment, MULTIG and DIVTIG, was assigned by grade level to half the remaining classes; and the one-game treatment, DIVTIG, was used with the other half of the remaining classes."

d) Although the Wilcoxon-T appears to be appropriately used, as only half of the 1976 classes were pretested, one must assume that of the data presented at the posttest level, only half of the data (those paired with pretest scores) were used to calculate the Wilcoxon-T scores. As not all readers of the article would be very familiar with the Wilcoxon matched-pairs, signed-rank procedure, it would have been helpful to include a sentence explaining the "half-use" of the 1976 posttest data.

A final area of comment is related to the "large" increase in speed test scores. The authors appear to be particularly impressed with the overall 56 percent increase in the 1977 speed scores. Yet this increase is obtained from three sets of classes. One, the fifth-grade classes, showed a 129 percent increase; the other two, combined fifth-sixth-grade classes, had only 23 percent and 27 percent increases. What factor(s) account for this great disparity? What factor(s) are operating within these fifth-grade classes that are not operating within the other classes?

This study is a positive contribution to this area of research, but should be considered with the following in mind: its lack of generalizability (to the whole school year) and its emphasis on overall average percentage score increases attributable only to the treatment effect.

Good, Ron. CHILDREN'S ABILITIES WITH THE FOUR BASIC ARITHMETIC OPERATIONS IN GRADES K-2. School Science and Mathematics 79: 93-98; February 1979.

Abstract and comments prepared for I.M.E. by JAMES M. MOSER, University of Wisconsin-Madison.

1. Purpose

The purpose was to determine the ability of young children to understand the operations of addition, subtraction, multiplication, and division from the point of view of sets of concrete objects.

2. Rationale

The author had written a previous position paper suggesting that the sequence of early mathematics instruction be changed in favor of interrelating the four basic operations. Beginning with concrete experiences, the difficulty level should then increase with number size rather than by type of operation. Several other researchers, including Piaget, were cited as having presented evidence suggesting that concrete, manipulative experiences should come first. Thus, the author sought to determine in the present study whether the four basic operations were accessible to youngsters within a concrete materials context.

3. Research Design and Procedures

Fifteen children each from grades K, 1, and 2 from the Florida State University Developmental Research School were individually interviewed during a 20-30 minute period, being given a set of seven tasks. The tasks included one-to-one correspondence, number conservation, counting, adding, subtracting, multiplying, and dividing. Subjects were given poker chips to use. A specially constructed number board depicting sets of chips circled with string and arranged in order (see Figure 1) was used with the last four tasks. Those tasks are listed below.

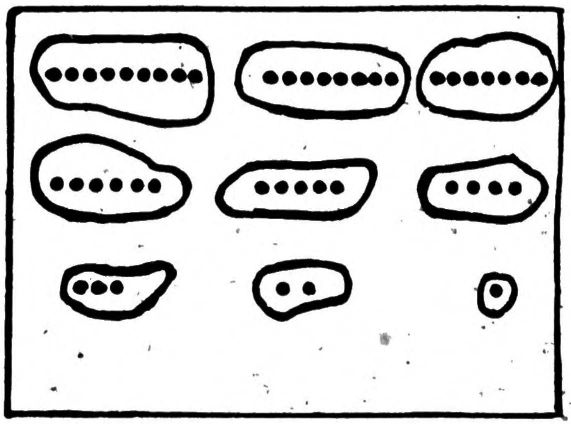

Figure 1. Number board used in the interviews, depicting sets of poker chips.

Task Four: Adding.

1. Do you see a group on the board just like my group (of poker chips)? How can you tell?
2. Do you see a group on the board just like your group of chips (child has eight chips)? How do you know?
3. Which group on the board do you need to add to yours to make mine? How do you know?
4. If you add this group of chips (three) to this group (four), which group on the board will be just the same? How can you tell?

Task Five: Subtracting.

1. How many chips do I have to remove to make this group (of chips) just like this group (on board)?
2. How many chips are in this group (six)? If I take these two away, how many will be left?
3. If I start out with this group of chips (seven) and I end up with this group (four on board), how many chips did I lose?

Task Six: Multiplying.

1. If I give you two groups like this (set of 3 chips), how many will you have?
2. If I give you three groups like this (3 chips), how many will you have?
3. If I give you four groups like this (2 chips), how many will you have?

Task Seven: Dividing.

1. Can you make two equal groups out of this group (six)?
2. Can you make two equal groups out of this group (seven)?
3. Can you make three equal groups out of this group (six)?
4. Can you make three equal groups out of this group (eight)?

For each task, questions were asked using both poker chips and the number board, until the interviewer was satisfied that each child had indicated his or her best ability to respond.

4. Findings

All 45 students were able to count to 10 and establish one-to-one correspondence using the poker chips. Table 1 shows data for number conservation and Table 2 presents data for the four tasks dealing with the basic operations.

Table 1

Conservation of Number Task

	Conserver	Transitional	Nonconserver
Grade K	6	4	5
Level 1	8	4	3
2	14	1	0
	28	9	8

Table 2

Students' Success Tasks 4-7 by Grade Level

	Conserver			Transitional			Nonconserver		
	K	1	2	K	1	2	K	1	2
Task 4 (Adding)	6	8	14	4	4	1	5	2	0
Task 5 (Subtracting)	6	8	14	3	4	1	5	2	0
Task 6 (Multiplying)	1	6	14	1	2	1	0	1	0
Task 7 (Dividing)	2	4	14	0	2	1	0	1	0

Most children categorized as transitional or nonconservers tended to rely exclusively on counting in responding to questions in Tasks 4 and 5.

5. Interpretations

Counting can be used as a successful procedure in dealing with simple addition and subtraction problems when objects are present. The sharp drop in success on Tasks 6 and 7 for children in grades K and 1 can be explained by their reliance on perception and/or counting as means for solving number-related problems.

Making simple classes and series, conserving number, and using reversibility all begin to appear at about 6 to 7 years of age and it is not until this point that children can get beyond counting as a means to solve "operations on numbers" problems. By second grade it appears that most children have a concrete, intuitive concept of the operations of addition, subtraction, multiplication, and division. If multiplication and division are dealt with in the context of manipulative materials, it appears that children who are able to conserve number can understand these operations much as they conceptualize addition and subtraction.

Much of the kindergarten and first-grade mathematics curriculum should consist of concrete activities related to classification, seriation, conservation, and reversibility. When logical operations are introduced to children who are developmentally ready to understand their meaning and usefulness, all four operations should be interrelated. Since place value is one of the most difficult concepts for children to understand, multi-digit numbers should be avoided until the children show that they understand the four basic operations.

Abstractor's Comments

I would like to make some comments about the tasks, their administration, and scoring. In the one-to-one correspondence task, the author states that 8 red and 9 white chips were used to determine ability to establish one-to-one correspondence between the sets. How can that be when no such correspondence exists between two sets of unequal size? Next, the counting task (tasks?) was characterized as determining ability to count up to ten. How was this done? Was simple rote counting the criterion or was the attempt to look at "rational counting?" Without belaboring the point, was the counting task made up of more than one

sub-task as was the case with the four operations tasks, and if so, how was it scored? In the same vein, we do note that the four operations tasks were made up of several sub-tasks. How was success rated --if a child passed one, two, or all three parts? On a much higher level of concern, I feel that these short tasks are very superficial measures of whether a child can deal with the basic operations, much less exhibit any understanding of those operations.

The almost perfect success rate on the addition and subtraction tasks by the subjects from grades K and 1 deserves mention. It is practically unbelievable, especially when one looks at the difficulty of the tasks involved. Sub-tasks 1 and 3 in subtraction are well known and documented in the literature as being quite hard for children. The fact that kindergarteners and first graders did so well is astonishing. I am led to suspect one or both of two possibilities exist. First, the so-called "questions" by the interviewer during the task session constituted a large measure of teaching and coaching. Second, the subjects were extremely atypical. This may well be the case when subjects are drawn from a university-based laboratory school. In either instance, the results are open to serious question in terms of generalizability.

Finally, I want to express my concern over yet another example of a far-too-often-used practice, namely drawing conclusions and inferences not based upon the evidence. I have no real quarrel--in fact, I support on an intuitive and theoretical basis--the author's suggestion that we reconsider the sequencing of the elementary mathematics curriculum in terms of interrelating the four basic operations. But the fact that 15 probably atypical second graders succeeded on some superficial tasks purportedly measuring understanding of and ability with those four operations does not imply that the curriculum should interrelate them. And certainly the results have nothing at all to say about place value and its potential difficulty.

In short, this article contributes very little to what we know about how children can and should solve problems in arithmetic.

Greenstein, Jane and Strain, Phillip S. THE UTILITY OF THE KEY MATH DIAGNOSTIC ARITHMETIC TEST FOR ADOLESCENT LEARNING DISABLED STUDENTS. Psychology in the Schools 14: 275-282; July 1977.

Abstract and comments prepared for I.M.E. by J. DAN KNIFONG, University of Maryland.

1. Purpose

The study attempts to judge the suitability of using the Key Math Diagnostic Arithmetic Test (Key Math test) as a diagnostic tool with learning-disabled adolescents.

2. Rationale

The identification of learning-disabled (LD) children has often relied on linguistic ability. One possible reason for this orientation is the lack of a suitable mathematics test. The Key Math test seems a likely candidate for both the identification and description of a child's learning disabilities. Specifically, would the Key Math test identify LD adolescents through performance means, subtest performance patterns, or computational error patterns which might then be correlated with the child's years behind grade level? And, would the test also serve to describe individual children's difficulties with mathematics?

3. Research Design and Procedures

Eighty-two adolescents (74 boys and 8 girls) between the ages of 12 and 17 who had been identified as learning disabled were given the Key Math test. Each adolescent was at least two years behind grade level and had been identified by a school psychologist as being learning disabled. Although several different tests were used by the psychologists, each adolescent had been tested on the WISC-R or WAIS and Bender-Gestalt. The Key Math subtest patterns and response errors from these students were then compared with those of a normal population. A factor analysis of the subtest scores was performed and the correlations between the means of each subtest score and years behind grade level were computed.

4. Findings

The LD pattern of scores of the 14 subtest nearly matched those of a normal fourth-grade population. This was interpreted as a "peaking" effect in the LD adolescent's ability, probably due to the abstract nature of mathematics beyond the fourth-grade level.

Statistically significant Pearson Product Moment Correlation Coefficients ranging from -.18 to -.44 were established for 11 of the 14 subtests vis-a-vis years behind grade level. The three correlations which were not significant were explained in terms of the content of the subtests.

The fourteen subtests are grouped by the developers as covering three major content areas: Content, Operations, and Applications. However, a factor analysis using a verimax rotation indicated only two major factors: "Operations" and "Applications." The third subtest grouping, "Content," split between these two factors. This result was interpreted as suggesting that the original topical categories were inappropriate for working with LD children.

Five computational error patterns were identified and the frequency of their occurrence among normal and LD children was compared. Only one of these patterns (the faulty recall of a number fact) was more frequent among the normal group.

5. Interpretations

It was concluded from the Key Math test that LD adolescents:

a. function at the concrete level;
b. cannot do problems requiring "chained" operations, e.g., if $3+4 = \square$, then $\square + 6 =$ ______; and
c. follow a specific pattern of computational error.

It was further concluded that although the factor analysis does not support the three-part categorization of the subtests proposed by the developers, the test is nevertheless a useful diagnostic instrument for LD children.

Abstractor's Comments

Several of the interpretations and recommendations are not supported by the scope of the study or the data.

Since specific criteria are not offered for determining "concrete level functioning," there is no basis for claiming that the test responses indicated these adolescents were at that level.

It is interesting that the LD adolescents "peaked out" at the fourth-grade level, but it is not clear whether this is attributable to the abstract nature of the later mathematics or some other cause.

The fact that a factor analysis does not support the three content areas of the test as proposed by its developer may indicate that these categories are inappropriate for LD children, or it may indicate that this sample tended to do about as well on the third category as they did on the other two. A careful analysis of the test content would be necessary to answer this question.

The only specific computational error pattern noted for these adolescents was that they made fewer number fact errors than the normal group and that they made more of the other five types of errors than the normal group.

In the context of this study there are two separate, but related, projected uses of a diagnostic mathematics test. Such a test should identify LD children on the basis of their mathematics performance and describe individual student weaknesses as an aid to planning instruction. Clearly, adolescent LD children score very low on this test and their low score correlates with their years behind grade level. However, the study does not address the question of whether there are other, non-LD children who score equally poorly on these tests. This would seem to be a crucial question if the test is to be used to identify LD children. The second projected use of the test rests on its content validity, an issue not addressed by the study. Whether the Key Math test, or any other test, actually includes a balanced range, type, and difficulty level of problems covering a particular grade level of the mathematics curriculum can only be determined by a systematic analysis of those items. Such an analysis may or may not reveal the Key Math test as comprehensive and thorough.

Kieren, Thomas E. and Nelson, Doyal. THE OPERATOR CONSTRUCT OF RATIONAL NUMBERS IN CHILDHOOD AND ADOLESCENCE--AN EXPLORATORY STUDY. Alberta Journal of Educational Research 24: 22-30; March 1978.

Abstract and comments prepared for I.M.E. by LESLIE P. STEFFE, The University of Georgia.

1. Purpose

Kieren and Nelson explored the view of rational numbers as operators held by children and adolescents. Five questions were investigated:

a. Is there a change over age of reaction to operator settings?

b. Do there appear to be stages in this development?

c. Will there be difficulties with the concept of inverse in these settings?

d. Are there differences in reaction to unit as opposed to non-unit fractions?

e. What mechanisms do children and adolescents use in handling these settings?

2. Rationale

While formal instruction with whole numbers has been focused on computation, it is based on a large amount of informal and practical experience. Instruction with rational numbers has been based almost solely on a computational construct. There is little relationship to a body of experience of children or to the broad practical spectrum. The study is based on a much broader construct of rational numbers which has, as its capstone, mature functioning in a spectrum of rational number settings. Such functioning includes a well-developed concept of equivalence experience with five basic subconstructs, and mechanisms by which persons deal with the meaning of rational number situations.

3. Research Design and Procedure

Forty-five subjects were randomly selected, 5 in Grade 4 and 10 in each of Grades 5, 6, 7, and 8. Each 10 consisted of five females and five males. No more than two subjects came from any one classroom. Each subject was interviewed on a one-to-one basis using an apparatus

called a "packing machine," which packaged sheets of paper. Six tasks were designed and administered as a power test in the following order: a "1/2" task followed by "2/3," "1/3," "1/3 x 1/2," and "3/4 x 1/2" tasks.* Because of performance, all subjects did not complete all tasks. On any task, a subject could be faced with five direct and five inverse items. For example, on the "1/2" task, the numbers of input sheets were 16, 10, 8, 18, and 6. Each interview followed a set protocol, adapted to insure clarity. The machine was introduced and tried out to see how it worked, using up to six trials with feedback. The subject merely counted the input and output. Five prediction trials followed the initial trials, where the subject was given the input and was asked to record the output. Reasons were requested for several examples and a request was made for the subject to tell how the machine worked. Following this, five prediction trials were given, where the subject was given the output and was asked to record the input. Reasons and a characterization of the working of the machine were again requested. After the subject was unable to make predictions or understand how a machine functioned, a partitioning strategy for running the machine was taught. The subject then "ran" the machine and the experimenter had to answer questions.

4. Findings

a. The means for four age levels were as follows: 9.4 for subjects less than 11 years of age; 15.5 for subjects between 11 and 12; 28.9 for subjects between 12 and 13; and 31.3 for subjects greater than 13. Differences of the means were significant between the first and second age groups and the second and third age groups.

b. Six descriptive categories of responses were established: I, Mastered no tasks; II, Mastered "1/2" tasks (direct and inverse); III, Mastered unit tasks ("1/2" and "1/3" tasks); IV, Mastered unit and composition; V, Mastered unit, non-unit, and composition; VI, Mastered all items using proportions. Attaining four of five items in a task constituted mastery.

*A "3/4" task was administered also, but the report does not indicate its place in the sequence.

c. Of the first age group, 8 of 12 subjects were in category II, 3 were in I, and 1 was in III. Of the second age group, 6 of 10 were in II, 2 were in II, and 2 were in I or IV. Of the third age group, 3 of 11 were in V, 3 were in IV, and 5 were in II or III. Of the fourth (and highest) age group, 1 of 12 was in VI, 2 were in V, 3 were in IV, and 5 were in II or III.

d. Most responses were either correct (81) or incorrect (58) on both items of a pair of direct and inverse items. Only one response was incorrect on a direct item and correct on its corresponding inverse. Thirteen responses were the other way.

e. Almost one-half of the response were correct for the unit items but incorrect for the non-unit items. Only one response was the other way. Fifteen out of 104 responses were correct for both.

f. Students thought subtractively and not multiplicatively in analyzing problem settings--for the "2/3" operator, ($12 \rightarrow 8$) would be interpreted as "it's subtracting 4," and ($30 \rightarrow 20$) as "it's subtracting 10," not focusing on the multiples involved.

g. Over 91 percent of the subjects mastered the "1/2" tasks. Students would give "1/2" responses when confused on other machines.

h. Twenty-seven subjects learned to "run" the machine, interchanging roles with the interviewer--all subjects Grade 6 or above and 6 of 15 subjects below Grade 6. In all cases, only the direct aspect was learned with very limited carryover to inverse and composition tasks.

5. Interpretation

The authors note that the data indicate strong age and stage development of students' reactions to operator interpretation for rational numbers. Three levels of development are hypothesized: (a) a "1/2" oriented level where a child's fractional conceptualization is dominated by "1/2"; (b) a transitional level where subjects can handle units and composition of unit operators; and (c) a level where all forms of operators are handled. A fourth level is conjectured where operators are incorporated into a system.

The authors note also that thinking of operators as ratios is not necessary for achieving at the third level noted above. Only one

student, a 13-year-old, indicated use of proportions in analyzing the machines--she named the composition of "3/4" and "1/2" as "3/8" after one trial. Other students used a partitioning mechanism to get at non-unit operators.

Abstractor's Comments

The authors are to be commended for conceiving and carrying out this experiment. It represents a very difficult and, correspondingly, an understudied area of research in mathematics education. There are (at least) three critical achievements in a child's mathematical life--the idea of ten as a unit, the idea of fraction, and the idea of an unknown. Because these three ideas are so critical to the mathematical progress of children, empirical work done in an attempt to elucidate children's conception (and acquisition) of them must be both encouraged and, at the same time, widely discussed. It is in this spirit that my comments are made.

My first comments pertain to the claim that the data indicate stage development for operator interpretations. Very careful consideration must be given to the nature of developmental stages. Stages carry with them the requirements of discontinuity of behavioral organization, invariant sequence, hierarchical relations between successive stages, structural organization, and preparation and achievement periods. In view of these requirements, the authors rightly hypothesize three <u>levels</u> of development of the operator interpretation. But, what are the criteria for <u>levels</u>? Levels seem to involve measurement--a situation where one level being of higher rank than another a priori demands measurement. But the "levels" suggested do not seem to be of the nature of just measurement sequences. They are much more like stages! Why the dilemma?

The data are cross-sectional and are not concerned with the acquisition of operator interpretation of fractions--only with how children react to interviews. Piaget's stages were never with reference to more advanced mathematics--they always dealt with rudimentary concepts--and with good reason. To unearth stages with regard to more advanced mathematical concepts than Piaget studied requires intensive study of the dynamics of acquisition of particular concepts--much more detailed and

intensive study than interviews allow. Interviews can be highly suggestive and are not to be discouraged. But one cannot observe possible processes of change without a chance to understand what those processes might be. For example, how is "level 1" functionally integrated into "level 2"? Does "level 2" develop and then reorganize "level 1" or is "level 2" an abstraction of "level 1"? While other important questions need to be asked and considered, it is more important to note that research methodology used in mathematics education needs to be expanded and changed to be consonant with the intentions of the investigators.

My second set of comments pertain to the claim that thinking of ratios as operators is not necessary for achieving a level where a child can handle all forms of operators. I am not disputing that the 13-year-old child used proportional reasoning when she named the composition of "3/4" and "1/2" as "3/8." Even though she could have simply found the product of 3/4 and 1/2, these authors know proportional reasoning when they see it! However, because other students used a partitioning mechanism to get at non-unit operators does not mean they were not <u>capable</u> of proportional reasoning. The question is whether structures which allow a task to be assimilated are the same ones that an observer thinks he "sees" the subject using to execute a task. It is entirely possible that whole-number knowledge was brought to bear to solve a task which was conceptualized by virtue of proportionality. As no case was made by the authors that the children could not engage in proportional reasoning, their claim should be recast as a conjecture.

Again, my comments are meant only to extend discussion on what are to me very important issues raised by the experiment. They are not to be thought of as being derogatory of the study, as it was well done.

Kurtz, Barry and Karplus, Robert. INTELLECTUAL DEVELOPMENT BEYOND ELEMENTARY SCHOOL VII: TEACHING FOR PROPORTIONAL REASONING. School Science and Mathematics 79: 387-398; May-June 1979.

Abstract and comments prepared for I.M.E. by LARRY SOWDER, Northern Illinois University.

1. Purposes

   a) "A brief teaching program, suitably designed, can enable ninth and tenth grade students in prealgebra classes to become proficient in proportional reasoning.

   b) "A form of this teaching program making use of manipulative materials will be more effective than a form using pencil and paper activities only.

   c) "Student attitudes toward this teaching program will favor the form with manipulatives over the pencil and paper form." (p. 387)

2. Rationale

Several status studies, built on Piagetian work with proportional reasoning, have indicated that many adolescents have difficulty with problems involving proportions. Thus, whether instruction can improve students' performance on such tasks deserves investigation. Laboratory approaches "invite the active participation of all students, form a concrete basis for abstractions, and facilitate peer group interactions" (pp. 387-388). Hence, studying the effect of a laboratory approach seemed desirable.

3. Research Design and Procedures

There were two series of investigator-designed and piloted lessons, one involving manipulatives and the other involving only paper and pencil. In these lessons the students worked with constant ratio, constant sum, and constant difference relationships from tables of data, with laboratory settings (it is not clear that the manipulative version and the paper-pencil version involved the same settings), and with solving proportion problems. The lessons required 14 class periods.

Eight intact prealgebra classes, four for the manipulative version and four for the paper-pencil version, served as subjects. Four teachers were involved, each teaching each version to one class; these teachers underwent a 16-hour training course on the lessons. One class from each

of six other teachers provided a control group. All the classes came from four schools.

Equivalent forms of investigator-written, eight-item tests were given as the pretest, the posttest, and the (two-month) delayed posttest. Scoring of the items was based on "level" of response--a seven-category system describing the process of solution and documented in earlier studies. The upper levels reflect increasingly sophisticated forms of proportional reasoning. Attitude data were based on an attitude survey and observations.

The analyses reported were based largely on single items from the first four items on the test. Two of the other items were on secondary objectives (e.g., drawing graphs), but no reason was given for ignoring the remaining two items, except that they were similar to two of the items reported.

4. Findings

Roughly two-thirds of the students in the experimental classes exhibited proportional reasoning on the posttest, whereas only about one-sixth did on the pretest. The corresponding change in the control classes was from about one-tenth to one-third. Similar results held for the delayed posttest with the experimental groups.

5. Interpretations

a) "This study has shown that it is possible to advance the use of proportional reasoning of many secondary school students by means of a well-designed teaching program requiring approximately three weeks of school time" (p. 396).

b) "... the cognitive gains were equal for the two experimental groups" (p. 397).

c) "The attitude surveys and classroom observations made during the study ... showed that the manipulative version was considerably more popular than the pencil and paper version. ... We conclude that the manipulative laboratory materials had a motivating value for many students and should therefore be considered in secondary school mathematics classes when student interest and attitude toward the subject are important issues" (p. 397).

Abstractor's Comments

The status studies cited demand that we see whether instruction can improve students' use of proportional reasoning. I particularly liked the authors' choice of subjects and the use of a two-month delayed post-test. If long-term improvement can be accomplished with relatively unscholarly students, we teachers can take heart. On the other hand, some experienced teachers might say, "Why didn't they ask me whether it could be done? I've been doing it for years!" Now there is documentation.

It is not clear why the analyses centered on only four of the six relevant test questions. An authors' comment on that point, and on their feelings about the possible effects of the lack of random assignment, might have been comforting.

As the authors note, this study does not shed any light on which facets or combinations of facets of the lessons--e.g., laboratory activities, or the focus on the relationships possible among variables--might be responsible for the learning. Max Bell has described many educational studies as being at the "earth, air, fire, and water" stage of knowledge. I would so classify this study. (The investigators might disagree, since they used a Piagetian-based approach in developing the lessons.) In any case, such studies can be valuable, particularly when they suggest "molecules" that might be basic ingredients in learning.

Mayer, Richard E. DIFFERENT RULE SYSTEMS FOR COUNTING BEHAVIOR ACQUIRED IN MEANINGFUL AND ROTE CONTEXTS OF LEARNING. Journal of Educational Psychology 69: 537-546; 1977.

Abstract and comments prepared for I.M.E. by THOMAS R. POST, University of Minnesota.

1. Purpose

This series of three experiments was designed to test the hypothesis that different underlying rule systems (cognitive objectives) can be acquired even though the same levels of mastery performance on a given behavioral objective are achieved. These studies attempt "to specify more clearly the characteristics of different learning outcomes capable of supporting the same criterion behavior and to test more clearly the idea that different instructional conditions can lead to the acquisition of different outcomes supporting the same behavior" (p. 538). Examined is "counting behavior" in base three using both numbers and letters to represent the patterns.

2. Rationale

In this study behavioral objectives of instruction which focus primarily on the behavior produced by the learner are distinguished from "cognitive objectives" which specify "the psychological processes and structures that are sufficient to produce the needed behaviors" (Greeno, 1976, p. 123). Wertheimer (1959) has argued that concepts learned through meaningful apprehension of relations has a potentially broader learning outcome capable of superior transfer and long-term retention despite the fact that a less meaningful approach might produce the same level of performance on an immediate mastery test. Similarly Scandura (1970) and Ehrenpreis and Scandura (1974) have suggested that the same mathematical skill can be supported by a set of simple associations (low level rules) or by a higher order rule system capable of generating those associations.

An early evaluation of Sesame Street (Bogatz and Ball, 1971) indicated that the program has been demonstrated to be highly successful in imparting a number of basic cognitive skills such as labeling and rate counting, but "has been much less effective in teaching more complex rule-governed behavior" (Henderson et al., 1975, p. 480).

These and other results encourage the search for instructional procedures which might be responsible for producing broad and narrow rule systems in learning a mathematical skill such as counting.

3. Research Design and Procedures

In this series of three experiments, 68 subjects in introductory psychology at the University of California, Santa Barbara, learned to recite the first 12 or 18 responses in one of two formats in base 3. A meaningful context of learning was defined as a way to relate the counting to past experience with base 10.

Experiment 1: All 24 students in this study learned the same counting series, but for half the format was numerals (0, 1, 2, 10, 11, ..., 200) and for half it was letters (w, d, r, dw, dd, ..., rww). The numerals or letters were presented one at a time using a shielded typewriter. One transfer test consisted of four addition problems involving no carrying and four problems involving carrying (for each format); a second transfer test (on counting) required students to supply the next 10 items after a given numeral or letter.

Subjects were randomly assigned to treatment; Group Number received only number symbols during learning and transfer; Group Letter received only letter symbols. Each subject was instructed by the experimenter to type on the tape what he or she thought came next after the given symbol. All subjects continued until correctly anticipating all symbols in the list twice in a row. Transfer lists were then administered.

Experiment 2: The procedure was identical to that for Group Letter in Experiment 1 except that half of the 24 subjects (drawn from the same population as in Experiment 1) were shown the letter-to-digit conversion card prior to learning (Group Before) and half were shown the same card after learning but before the transfer test (Group After).

Experiment 3: This experiment refined the procedures of Experiment 2 by using a more homogeneous subject population (SAT scores averaged 600), by using a shorter counting sequence (13 items), and by controlling the time spent on each response (10 seconds). Items were presented via a slide projector (onto a screen) to two or three subjects, who wrote responses on paper. Only letters were used; the Before and After instructions indicated to subjects that they could "think of the letters as

numbers, w = 0, d = 1, r = 2, if that helps you."

4. <u>Findings</u>

<u>Experiment 1</u>: The 18 responses for 1 through 200 (or d through rww) were divided into six blocks with three positions within each block. (The positions were items ending in 1 or d, 2 or r, and 0 or w.) Subjects in Group Letter produced an average of 79.5 errors in learning to criterion, compared to 9.8 errors for Group Number [$F(1,22) = 44.29$, $p < .001$]. This result was expected due to the fact that <u>Ss</u> had more experience in counting with an ordered set of numbers than random letters.

The Group x Block Interaction was examined to determine if the groups had in fact used different strategies in learning the sequences. If so, Group Letter might produce a serial position curve similar to that obtained with nonsense syllables -- a strong primary effect and a leveling off -- while Group Number would not. The significant Group x Block Interaction [$F(5,110) = 8.14$, $p < .001$] resulted from the fact that Group Letter, as hypothesized, did perform better on the early blocks, while Group Number showed the reverse trend. The average per-pupil errors for Group Letter were 7.2, 12.6, 13.8, 15.0, 15.9, and 15.0, for Blocks 1 - 6 respectively. The corresponding errors for Group Number were 2.0, 1.6, 1.8, 1.6, 1.6, and 1.2.

A second prediction is that Group Letter should have approximately equal difficulty for base change and nonbase change positions, since all are equally meaningless, but Group Number should have relatively more difficulty with base changes than nonchanges. This prediction is partially consistent with the reliable Group x Position Interaction [$F(2,44) = 11.43$, $p = .001$], in which Group Number had relatively more difficulty with base change than nonchange positions, but Group Letter tended to have a more balanced pattern of errors. The average errors for Group Letter were 25.2, 19.2, and 35.4 on the two nonbase change positions and one base change position, respectively, and the corresponding errors for Group Number were 1.8, 0.8, and 7.2. It was noted that since Group Number outperformed Group Letter in all positions, the above interaction is not disordinal; however, an analysis of the percentage of errors to total errors shown in the left portion of Table 1 revealed that Group Number committed a higher proportion of errors at the base change position than Group Letter

[$t(22) = 6.12, p < .001$]. The difference in the proportion of errors at the base change is consistent with the hypothesis that different learning strategies were used.

Table 1

Proportion of Total Errors Occurring at Base Change Positions for Two Groups

Group	Experiment		
	1	2	3
Letter or After	.48	.45	.43
Number or Before	.78	.55	.61
t test, p	< .001	= .05	< .001

Note. Base change position ends with w or 0. Experiment 1 compares Letter and Number Groups; Experiments 2 and 3 compare After and Before Groups.

Since both groups learned the same sequence to the same criterion of mastery (two errorless trials), no difference in post-test performance was predicted. Because of differential error patterns, it was concluded that the groups <u>may</u> have used different learning strategies; i.e., <u>if</u> Group Letter <u>S</u>s learned by a rote strategy, the predicted outcome of learning would be a memorized chain of 18 sequential responses. <u>If</u> subjects in Group Number learned by relating the new sequence to their past experience with number systems, the predicted outcome of learning would be a system of rules for generating the first 18 items in base 3. Thus, it was predicted that Group Number would do better on transfer tasks, and this was borne out. Group Number performed better than Group Letter on arithmetic [$t(22) = 10.77, p < .001$] and on counting [$t(22) = 3.65, p < .01$] related post-tests.

<u>Experiment 2</u>: Group After (identical to Group Letter in Experiment 1) averaged 82.2 errors in learning to criterion, compared to 37.6 errors for Group Before [$F(1,22) = 4.71, p < .05$]. The pattern for Group After was similar to that of Group Letter in Experiment 1, with better performance in early blocks than middle or later blocks. The two interactions tested, Group x Block and Group x Position, occurred in the predicted di-

rection but were not significant ($p = .05$). Group Before performed significantly better on counting transfer test ($p = .05$), suggesting that Group Before acquired a broader learning outcome. Apparently the conversion table served as a meaningful context (see Table 2) of learning which allowed broader encoding of the to-be-learned sequence.

Experiment 3: As in Experiment 2, Group After committed significantly more errors than Group Before (50.0 vs. 17.5, $F(1,18) = 24.27$, $p < .01$). This time Group x Block Interaction [$F(3,54) = 6.38$, $p \leq .01$] reached statistical significance. Once again the Group x Position Interaction failed to reach statistical significance. Group Before's higher proportion of errors at base change portions was, however, significant [$t(18) = 3.37$, $p < .01$]. Group Before performed significantly better than Group After in the arithmetic test [$t(18) = 2.72$, $p < .02$] (see Table 2).

Table 2

Proportion Correct Response on Transfer

Tests for Two Groups

Group	Exp. 1		Exp. 2		Exp. 3	
	Add	Cou	Add	Cou	Add	Cou
Letter or After	.01	.18	.49	.61	.17	.21
Number or Before	.56	.72	.49	.86	.50	.61
t test, p	< .001	< .01	ns	= .05	< .05	< .02

Note. Experiment 1 compares Letter and Number Groups; Experiments 2 and 3 compare After and Before Groups.

The studies suggest that qualitatively different learning processes were used by the various treatment groups. In general, Ss in Group After (or Group Letter in Experiment 1) seemed to use what could be called a rote learning set. Ss in Group Before (or Group Number in Experiment 1) seemed to use a different strategy that could be called a meaningful learning set (i.e., relating presented material to an integrated set of existing knowledge and in particular relating to part-counting experience in base 10).

5. Interpretations

These results indicate that the same behavior (e.g., counting to 12 or

18 in base 3) can be supported by entirely different learning processes and learning outcomes. Further, these results caution against teaching a particular problem-solving behavior by using behavioral objectives for mastery that ignore the underlying cognitive objectives of instruction (Greeno, 1976). To avoid narrow learning outcomes such as exemplified by Group After in the present study, instructional objectives should be sensitive not only to what behavior is learned, but also to how it is learned and structured in memory. There are situations in which narrow learning outcomes might be preferred, but when new learning must build on the outcomes of present learning (such as in number systems), narrow learning would have particularly serious consequences.

Like some previous studies on the sequencing of an advance organizer, the present results argue against the straightforward idea that Group After and Group Before must have learned the same thing, since they were presented with the same stimuli. Broader learning of new technical skills may be enhanced by providing a meaningful context prior to learning rather than after, presumably because the context serves to establish a meaningful learning set (Ausubel, 1968).

Abstractor's Comments

The concern of rote vs. meaningful learning is not new. Bruner, Dienes, Piaget, Gagné, and others have all addressed this issue in one way or another. Each has advocated the development of coherent schemas or structures which relate past to pretest learnings and lays the foundation for the meaningful assimilation and accommodation of new experiences into one's existing cognitive map.

Mayer's three studies, like earlier investigations, conclude that "broader learning of new technical skills may be enhanced by providing a meaningful context prior to learning rather than after, presumably because the context serves to establish a meaningful learning set." Hardly an idea foreign to mathematics educators, it is nonetheless of crucial importance. This paper offers additional evidence to support the existence of these differential cognitive structures.

Mayer, in this series of three related experiments, has inferred certain aspects about the nature of the underlying cognitive structures which support overt behavior -- an admittedly difficult area. These

inferences, perhaps by necessity, are based on what might be thought of as circumstantial evidence. At some points in this paper the author appears to be arguing the converse of an admittedly true statement. That is, given that the statement "if X Then Y" is true, the converse "if Y then X" is believed to be true. This may or may not be the case. Mayer has established in several cases by past research that X does, in fact, imply Y. A potential difficulty arises when one concludes from this that "if Y then indeed X." It must be pointed out that Mayer is careful to qualify his conclusions with the use of "if," "might," "could," etc. The danger of overly ambitious conclusions is present, however, unless the reader is careful.

Mayer's use of college-age students with "no prior experience with counting in nonbase 10 number systems" may have changed the perceived nature of the problem task from that of a counting-related problem to that of a problem more concerned with pattern recognition. This research, therefore, may not really be dealing with counting behavior as suggested.

The author's distinction between rote and meaningful context of learning is in need of further consideration. Mayer defines meaningful context as "providing a way to relate the counting series to past experience with base 10, e.g., presenting the letter to digit conversion list prior to learning." A rote context is presumably the absence of this.

The fact that subjects might, in fact, exhibit a variety of strategies in attempting to deal with the "counting tasks" seems to have been overlooked. Mayer suggests that students will relate the task to past experience with base 10 and that the crucial issue (in Experiment 2 and 3) is whether or not Ss are presented with conversion (letter to number) information before or after instruction. An alternate possibility exists. It is indeed possible to address successfully the counting issue without recourse to the structural properties of base 10 or numeration systems in general. When faced with the sequence w, d, r, dw, dd, dr, rw, ..., it is possible to predict the next element in the sequence by simply observing the patterns which emerge. It would also be feasible to use the same procedure if the numbers 0, 1, and 2 replaced the letters w, d, and r, respectively. Several individuals, when asked by this reviewer to predict the next number (letter) in the sequence, responded in this manner; i.e., not on the basis of previous knowledge of base 10 but by search-

ing for recurring patterns, be they number or letter. This point is made only to suggest that alternate solution strategies exist for the problems solution in much the same way that real problems usually elicit various solution strategies from a variety of individuals. This then casts some doubt on the appropriateness of Mayer's definition of meaningful context. The assumption that the presentation of a numerical sequence is somehow more meaningful than the presentation of a letter sequence because it alone can be related to previously established cognitive structures (base 10) is questionable. An alternate hypothesis would suggest that meaningful context in this case implies a good deal more than familiarity with the base 10 numeration system; i.e., the ability to generalize a pattern given a small number of examples. It seems possible that, for a given individual, either context (letter or number) could constitute a meaningful context given appropriate previous experiences.

It would have been useful and relatively easy for the author to have interviewed Ss in these experiments to ascertain the nature of the strategies employed. It is unfortunate that this was not done. Some information was gathered regarding solution methods used, but this was not reported in the article.

This raises the question as to why individual students given the number sequence outscored those given the letter sequence in Experiment 1 and those given the conversion suggestion (i.e., let w = 0, d = 1, and r = 2 in Experiments 2 and 3), if it were not because of direct relatedness of the sequence to previously developed cognitive structures? The answer may lie in the domain of the perceived relatedness of the context within which the problem is embedded, not in the meaningfulness of the problem itself. Subjects are generally more comfortable with what seems to them to be a more concrete embodiment of the problem situation. Having had more experience with number related patterns, it would seem likely that subjects would be more comfortable in this area. This is analogous to the uneasiness which first-year algebra students face when first confronted with the concept of variable.

It seems necessary to distinguish meaningful context from meaningful learning.

The difficulties resulting from the fact that both groups in Experiment 1 received different posttests was rightfully acknowledged by Mayer.

He suggests (and I agree for the reasons just discussed) that letters may be inherently more difficult regardless of the experience. It would have been interesting to have each group take the others' posttest as a measure of transfer. If the Number Group really did develop a higher order rule system, as suggested, it should be able to deal effectively with the letter sequence with or without the letter to number conversion suggestion. That is, it should be able to deal comfortably with structural similarities (isomorphisms) between the number and letter patterns.

It is useful to re-affirm continually the importance of meaningful learning experiences in mathematics education. Overly conservative trends such as the Back-to-the-Basics movement have a tendency to ignore or sublimate this aspect of instruction to the more visible and easily measured overt performance on clearly specified behavioral objectives. Unfortunately, those objectives most easily measured are precisely those which reflect algorithmic procedures. These, in turn, can be taught in either rote or meaningful ways. All too often the rote is employed out of proportion to its importance. Mayer is correct in urging us to consider dimensions of student learning beyond that implied by mastery of behavioral objectives. "Instructional objectives should be sensitive not only to what is learned but also to how it is learned and structured in memory." This is a much more difficult assignment.

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Abstract and comments prepared for I.M.E. by JANE SWAFFORD, Northern Michigan University.

1. Purpose

The purpose of the study was to explore the possibility that the absence of significant differences in comparisons of school or program effectiveness might be due to varying teacher effectiveness within the schools or programs. Specifically, the study sought to determine what proportion of the variance not accounted for by between-schools differences could be attributed to differences between-teachers-within-schools rather than to individual student differences on *error* variation.

2. Rationale

Large-scale studies of program effectiveness have most often led to the conclusions that different programs have little differential impact since within-school (or within-program) variation is largely relative to between-school (or between-program) variation. However, it is noted that in many of these studies, the data have been aggregated across teachers within each school or program. Hence, variance among teachers within each school is assigned to individual pupil or *error* variation. This study was designed to investigate the wisdom of ignoring teacher effectiveness in assessing program or school effectiveness.

3. Research Design and Procedures

Data from the First International Study of Achievement in Mathematics (IEA) were utilized in this investigation. From the American IEA sample, a subset of 108 schools at the 8th- and 10th-grade and 33 schools at the 12th-grade level were selected. Each school had two or more teachers who each had at least five students participating in the IEA study. Using the IEA data, variance estimates and percentages for school, for teacher-within-school, and for pupil-within-school-and-

teacher were calculated on measures of (1) mathematics achievement, (2) interest in mathematics, and (3) socioeconomic status.

4. Findings

On the measure of mathematics achievement, the within-school variance was somewhat higher than the between-school variance at the three grade levels, ranging from 60 percent to 75 percent of the total variance. However, the variance between teachers within school was found to account for approximately 25 percent of the total variance at the three grade levels and from 30 percent to 40 percent of the within-school variance. Hence, teachers-within-school did explain much of the variance in mathematics achievement.

On the measure of interest in mathematics, the within-school variance was considerably higher than the between-school variance, ranging from 83 percent to 92 percent of the total. However, the teacher-within-school variance was small and rather constant at each grade level at 6 percent to 8 percent of the total variation. Hence, teachers-within-school did not explain much of the variation in students' attitude.

On the measure of socioeconomic index, moderate between-school variations (22 percent to 25 percent of total) and large within-school variations (72 percent to 78 percent of total) were found. However, teacher-within-school variation accounted only for between 6 percent and 8 percent of the total variance. Hence, teachers-within-school did not explain much of the variation in socioeconomic status.

5. Interpretations

The study concludes that a sizable component of the variation traditionally defined as error variance in studies of school or program effectiveness can be associated with the differential effects of teachers-within-schools.

Abstractor's Comments

The use of existing data pools to investigate questions concerning research design and interpretations, unfortunately, has its perils.

This study both acknowledges and illustrates the problems. A study designed to answer one set of questions is sometimes poorly designed to address another. The IEA study was designed to assess international differences in student achievement, not teacher effectiveness. The sample was chosen to be representative of the student population in each country. Information concerning the identity of the student's teachers was incidentally collected. Information concerning the level of the student's mathematics course was not. Hence, rather than differences in teacher effectiveness, the observed differences between students with different teachers could reflect differences in the level of the mathematics course (e.g., General Mathematics versus Algebra II) or differences in ability grouping for the same course.

While one might suspect that within a school or program there can be wide variation in the effectiveness of individual teachers, this study does not substantiate it. It does, however, reiterate the caution against ignoring the teacher variable in program evaluation and suggest a research design for estimating the impact of that variable.

guidance was given only if there was complete frustration in the group. The control groups did the worksheets with the teacher directing the reading, answering the questions, and giving "direct assistance" to those with wrong answers.

The groups were compared on an observation checklist and comment sheet on:

1) whether or not they could complete the task, and
2) how the pupils worked affectively with each other.

The analysis consisted of comparing the number of groups that completed the slide-rule task without teacher assistance and the pupils' subjective comments.

4. Findings

1) All the experimental groups completed the slide-rule task; none of the control groups did.
2) Experimental groups helped and encouraged each other; control groups did not. Examples of the interactions are quoted in the article.

5. Interpretations

Students can be helped to work effectively in groups on mathematics problems. The author states: "Furthermore, while doing this, they are gaining self-confidence, interest in math, and a more healthy attitude toward their peers. ... Children's personalities bloom." This doesn't happen often "while they're learning mathematics."

Abstractor's Comments

1) No comparison was made on what content was learned in training tasks. Did those with group-oriented instruction learn more than those with teacher direction?
2) Teachers who are deciding on how to use classroom time need to compare the learning by students in well-trained groups with good, teacher-oriented, full-class instruction.
3) One rural class of 28 taught by six preservice teachers does not generalize to most classrooms.
4) The affective results quoted above in "Interpretations" need to be verified.

Schoenfeld, Alan H. EXPLICIT HEURISTIC TRAINING AS A VARIABLE IN PROBLEM-SOLVING PERFORMANCE. Journal for Research in Mathematics Education 10: 173-187; May 1979.

Abstract and comments prepared for I.M.E. by RICHARD E. MAYER, University of California, Santa Barbara.

1. Purpose

The author lists three main goals: (1) "to study the impact of instruction in five heuristics on some students' performance on a series of problems"; (2) "to see if other students working the same problems ... but not receiving the heuristics instruction would use or intuit the strategies"; and (3) "to see, by comparing the two groups, if explicit instruction in heuristics 'makes a difference'."

2. Rationale

There has been much written about how to teach students to become better problem solvers (such as Polya's classic How to Solve It and Mathematical Discovery), but such work is not closely connected to research and theory in the psychology of problem solving. Similarly, there has been much research and theory on problem-solving heuristics in cognitive psychology (such as Newell and Simon's Human Problem Solving), but little connection with practical issues in mathematics education. The present study attempted to bridge this gap by taking heuristics derived from cognitive psychology and studying their use in real-world mathematical situations. Thus, the tools of cognitive psychology--descriptions of the mechanisms underlying problem-solving performance--are applied to the problems of mathematics education. The result is an attempt to determine the "mechanisms" used by mathematical problem solvers and see if these "mechanisms" can be successfully taught.

3. Research Design and Procedures

Four subjects served in the experimental group and three subjects served in the control group. Subjects were experienced in mathematics and science, since they were all upper-division mathematics and science majors at the University of California, Berkeley.

All subjects took a five-problem pretest and a five-problem posttest, with a maximum of 20 minutes allowed for solving each problem. The problems included proofs, complex algebra story problems, and series sum problems. All subjects received written and tape-recorded instruction on how to solve 20 problems (including the five pretest problems) over a two-week period.

The independent variable was type of instruction: the experimental group was given a list and description of five useful strategies, and explicitly told that a given instructional problem should be solved by a specific strategy. For experimental subjects, all problems in an instructional session were solved by the same strategy. The control group was given the same set of 20 problems and the same written and recorded instruction, except that no list and description of the strategies was given, no explicit mention of the used strategy was given, and the problems solved in a session were not all of the same strategy type. The dependent variable was the change in score from pretest to posttest; in addition, thinking-aloud protocols were taken so that more detailed differences in performance could be determined.

4. Findings

The experimental group increased from an average of 20 percent correct on the pretest to an average of 65 percent correct on the posttest, while the control group averaged 25 percent on both tests. In spite of the extremely low sample size, the differences in change scores are significant. Analysis of protocols indicated that the control group did not tend to make efficient use of heuristics, while the experimental did for several types of heuristics.

5. Interpretations

The author concluded that

> a detailed look at the students' solutions to the test problems supports the results suggested by the statistics: Conscious application of a problem-solving strategy does make a difference. (pp. 182-3)

Moreover, students can transfer training, at least on some strategies, from practice problems to posttest problems. The inconclusive results

on other strategies suggests that more work is needed in selecting and teaching the strategies. However, "the fact that a student knows how to use a strategy is no guarantee whatsoever that the student will indeed use it" (p. 183).

Students intuit heuristics minimally. Therefore, students need to be taught strategies explicitly. Further research is discussed: (a) "loosening" the format to make it approximate real-world conditions, or (b) varying the heuristics being studied.

Abstractor's Comments

On the importance of being cognitive. This paper shows a great improvement over previous work on "teaching problem solving" because the author deals with the mechanisms underlying performance rather than raw empiricism. This line of research demonstrates that it is not necessary to return to earlier research techniques in which "method A" is compared to "method B," a choice for 'which is better' is based on posttest scores, and no understanding of underlying cognitive mechanisms is provided. It is a pleasure to see that the "raw empiricism" of the past is finally being laid to rest.

This paper is representative of the growing symbiotic relationship between cognitive psychology and mathematics education. The domain of mathematics learning and problem solving provides an excellent arena for development and testing of cognitive theory; and the analytic tools of cognitive psychology provide an excellent starting point for improvements in mathematics instruction. For example, the recent advances in our understanding of algebraic problem solving by people like Hayes and Greeno and of physics problem solving by Larkin and Simon are highly encouraging. Only mutual benefit can come from the natural give and take between cognitive psychologists and mathematics/science educators.

All you need is more. The introduction, method section, and results section of the paper--while clear and well written--all could be improved by providing "more of the same." The introduction seems to list related research work but makes no effort to describe the existing work in enough useful detail. The procedure allows for a nicely controlled study, and the study shows an overall level of tight

experimental control that is sadly lacking in many similar studies. However, the results are seriously limited by the fact that only seven subjects were used, and by the fact that all of them were already highly experienced in mathematics and science. Admittedly, the results are statistically significant--even when subjected to a t-test, which the author did not attempt. However, the importance of the finding demands that an additional replication be carried out before the author can make too many claims. Furthermore, less experienced subjects should be used, since it is they who are likely to be most in need of strategy training.

The same request for "more of the same" could be made with respect to the analysis of protocols. The author makes a nice start at describing the differences in protocols of subjects in the two groups. However, much more detail is needed. In particular, there is a need to tie the protocol analysis to some specific theory of problem solving. Analyzing protocols can become a trap in which the researcher has so much specific information that he or she cannot see any general patterns in the results. The present author has not fallen into this trap, but there is a need to become more concrete about the general patterns located in the protocols. A final problem with the protocol procedure is that "thinking aloud" is not a normal or natural way to solve problems. It is not possible to determine how this procedure influenced the "number correct" data for the posttest nor is it possible to disentangle the effects of "thinking aloud" from the effects of the experimental treatment.

Can we teach problem solving? This paper deals with an old question --Can we teach problem solving?--and deals with it in a creative way. However, there are many pitfalls in "training of problem solving" research. First, there is the problem of defining problem solving. This is rarely done. The goal of instruction for problem solving should be clearly specified. There are many instructional procedures which go under the label of "teaching for problem solving," but they vary widely in content. We need an objective definition of what it is that we are trying to teach.

A second problem concerns the traditional usefulness of instruction in general problem solving. A typical finding concerning programs developed to teach creative problem solving to school children is that such programs have either little effect or that they improve problem-solving performance mainly for specifically similar problem-solving situations (see Mansfield et al., 1978). If this is also the case for cognitive techniques--the present experiment deals only with near transfer to very similar problems--this raises the question of whether it is useful to assume there are any "general" problem-solving techniques in humans independent of specific problem-solving subject areas.

References

Mansfield, R. S.; Busse, T. V.; and Krepelka, E. J. The effectiveness of creativity training. Review of Educational Research, 1978, 48, 517-536.

Silver, Edward A. STUDENT PERCEPTIONS OF RELATEDNESS AMONG MATHEMATICAL VERBAL PROBLEMS. Journal for Research in Mathematics Education 10: 195-210; May 1979.

Abstract and comments prepared for I.M.E. by GAIL SPITLER, University of British Columbia.

1. Purpose

The principal objective of the study was to examine (a) dimensions of verbal problem similarity perceived by students and (b) the relationship between a student's perception of problem similarity and performance on tests of verbal and mathematical ability.

2. Rationale

Polya has suggested that when one is solving a mathematical problem, it is useful to "think of a related problem." Silver argues that one's ability to use Polya's related problem heuristic is at least partially dependent upon the criteria which one uses in forming categories of problems. The study may be seen as an extension of the work of Chartoff (1977) and of Hinsley, Hayes, and Simon (1976). The article under review is a report of two studies, with the second being a refinement and replication of the first. The two studies will be discussed separately.

3. Research Design and Procedures (Study 1)

The sample consisted of 95 students in four intact eighth-grade mathematics classes taught by the investigator. All of the testing occurred during the normal mathematics class period. Measures of verbal and mathematical ability used were The Educational Records Bureau Mathematics Achievement Test (Mathematics Computation, Mathematics Concepts), Differential Aptitude Test (numerical ability, verbal ability, abstract reasoning ability), and Lorge-Thorndike (verbal and non-verbal IQ). The testing/treatment occurred over nine days. The first day involved the completion of the Card Sorting Task (CST) developed by the investigator. The CST consists of 24 verbal problems, on individual cards, which the student is asked to sort according to mathematical relatedness. The

problems varied systematically along two dimensions: mathematical structure and contextual detail. The students also supplied written explanations of their categories. On the second day the students attempted to solve problems 1-8 of the CST. On the third day the solutions to problems 1-8 were discussed. The fourth/fifth days and the sixth/seventh days paralleled days two/three considering problems 9-16 and 17-24, respectively. The number of completely correct solutions obtained by each individual student was recorded as the Problem-Solving Task (PST) score. On the eighth day the students were asked to complete the CST again. On the last day the students completed the Problem Relations Task (REL) developed by the investigator. The REL consists of ten verbal problems similar to those included on the CST. The students were asked to read the first REL problem, think about how they would solve it, and identify the CST problem which was mathematically related. The procedure was repeated with each of the ten REL problems.

The CST yielded two scores: a dimensional association score (DAS) and a pure category score. For each dimension the DAS was obtained by counting the number of pairs of problems, related along the given dimension, that a student put into the same category. The pure category score was obtained by counting the number of instances of student-formed categories containing at least two problems related along the given dimension and no problems unrelated to the given dimension. The REL score was obtained by counting the number of instances in which the student correctly identified that the given REL problem was related to a CST problem along a given dimension.

4. Findings (Study 1)

Students used four criteria to categorize the CST problems: mathematical structure, contextual detail, question form, and pseudo-structure. The latter dimension referred to associations based upon the presence of a measurable quantity such as age, weight, or time. A major difficulty with the CST problem set was identified in that many of the problems were related along more than one of the four dimensions. Only the structure and contextual detail scores were used in further analysis. There was a marked increase in the structure DAS score and the pure category

score between sorts 1 and 2. For each of the two dimensions, Pearson product-moment correlations coefficients were used to compare performance on the CST and REL tasks. The results indicate significant ($p < .005$) positive relationships between the REL score for the structure dimension and the pure category score for each sort, as well as for the structure DAS score for each sort. For the context dimension significant positive correlations were found between the REL score and the context DAS score, but the correlations between the REL score and each of the pure context scores were not significant.

The results suggest generally positive correlations between high performance on the ability measures and sorting based upon structure, and negative correlations between ability measures and sorting based upon contextual detail. Pre-solution sorting on the basis of structure was significantly ($p < .005$) correlated with PST performance, even when the effects of verbal and nonverbal IQ were simultaneously controlled; however, the association was not significant when the effects of mathematics concepts knowledge and mathematical computational ability were controlled. Post-solution sorting on the basis of structure was significantly ($p < .05$) correlated with PST performance even when the effects of IQ, concepts knowledge, and computational ability were simultaneously controlled.

3. Research Design and Procedures (Study 2)

The sample for this study consisted of 58 eighth-grade students enrolled in one of three intact mathematics classes taught by the investigator. None of the subjects had participated in Study 1. The CST was revised in light of the findings of Study 1 so that few problem pairs would be related along more than one dimension. All other procedures were the same as Study 1 except that the REL task was not included.

4. Findings (Study 2)

The results of Study 2 support the findings of Study 1. There was an indication that all four dimensions were used for classifying; there was an increase in the number of structure associations between sort 1 and sort 2; there was a positive correlation between measures of ability and sorting according to mathematical structure, and correlations

between the PST and structural sorting were significant even when the effects of ability were controlled with the same limitations as noted in Study 1. In addition, Study 2 indicated a generally negative relationship between the tendency to sort problems on the basis of question form and performance on tests of nonverbal IQ, concepts knowledge, computational ability, and problem-solving ability. However, the presolution pure pseudo-structure score correlated positively with PST performance, but not with general mathematical ability variables.

5. Interpretations

The four identified dimensions used by the students to categorize the problems are similar to the dimensions identified in similar studies. The findings suggest that students view problem relatedness in different ways and that there exists a strong relationship between mathematical ability and the tendency to perceive the mathematical structure of verbal problems and to relate problems on the basis of perceived structural similarities. The findings concerning the use of pseudo-structure categories suggest that this category was used by good problem solvers during the initial sort and that some less able problem solvers increased their use of this category on the second sort. The need for more research in this area was noted.

Abstractor's Comments

The one contribution made by this study is the identification of four categorizing strategies; however, as the investigator acknowledges, one must be careful not to generalize these results, as the problem sets were carefully constructed to elicit the recognition of certain predetermined dimensions. Having acknowledged this contribution, I must also state that I have many reservations about the study, both in the conceptualization of the problem and in specific detail. These concerns are listed below.

1. Aside from the identification of the four categorization styles, I find the other results of the study self-evident. One of the major findings of the study is that the sorting of problems according to mathematical structure is positively correlated with various measures of

ability. In fact, would one not define ability and/or intelligence so as to include the ability to perceive structure in events or situations? Another major finding of the study is that the PST score positively correlated with structural sorting even when various measures of ability and knowledge were controlled. While it is always satisfying to obtain statistically significant results with such controls, this result also seems self-evident. Students who perceived the structure of a problem were able to solve it.

2. It is important to recognize that an underlying assumption of this study is that the recollection of a related problem is somehow related to the way in which said problem is initially categorized by the student. This may or may not be true.

3. The study could have been strengthened in one or all of the following ways:

a) by the inclusion of some measure of general information sorting. As the investigator points out, evidence is available from a wide variety of fields concerning the perception of commonality of stimuli. Many cognitive style constructs are at least partially defined in terms of the type of categorizing style employed by the subject. If this study had included some measure of cognitive style it might have shed some light on the question of whether the sorting behavior was unique to mathematics or whether it is a generalized behavior that appears in a variety of situations.

b) by the inclusion of some measure of the retention of the problems and the system by which it was categorized. This would have helped to address the problem of whether initial perceptions influence later recollection of problems.

c) by the inclusion of problems for which the students did have algorithms appropriate for the solution of the problem. The CST set included only problems for which the students did not have sufficient algorithms for the solution thereof. Do students sort such problems in the same way that they would sort solvable problems?

4. The report itself lacks detail in several critical areas. This may not be the fault of the author. It may be that the brevity and succinctness required for a journal article is such that the necessary detail cannot be included. If such is the case, one must raise questions about the purpose of such journals. Specifically the report lacks detail concerning the CST and REL.

a) With respect to the CST, there is no indication as to which specific mathematical structures were used, why certain structures were selected, how the students performed on the CST (frequency of use of various categories and whether students tended to use one strategy over the others), and how many problems in the revised CST actually overlapped along one or more of the dimensions. The investigator does give some clues concerning the frequency of use of the various dimensions, but more detail is needed.

b) There is no indication given as to why the REL task was not included in Study 2.

In summary, I must acknowledge the contribution made by this study but I have many reservations about the overall report. In fact, one wonders why the results of Study 1 were included at all, as Study 1 was more a pilot study for Study 2 than an independent experiment. I heartily concur with one of the investigator's closing comments that this study raises a myriad of questions for further research, but I must add that I doubt that it has answered very many questions.

Thornton, Carol A. EMPHASIZING THINKING STRATEGIES IN BASIC FACT INSTRUCTION. Journal for Research in Mathematics Education 9: 214-227; May 1978.

Abstract and comments prepared for I.M.E. by JOSEPH N. PAYNE, University of Michigan.

1. Purpose

The purpose was to explore the effect of teaching thinking strategies for basic facts in grades two and four.

2. Rationale

The results from studies done earlier by Thiele (1938) and Swenson (1949) were given as reasons to conjecture improvement in basic fact learning through the explicit teaching of strategies. Recent studies and recommendations by Rathmell (1978), Ashlock (1971), Hall and Trafton (1974), Jerman (1970), and Myers and Thornton (1977) were cited also. The author noted the current practice of not using strategies in mathematics classrooms.

3. Research Design and Procedures

Intact groups from multi-level classes in grades 2 and 4 were pooled to form two groups, experimental (E) and traditional (T). For E, n = 25 in Grade 2 and n = 23 in Grade 4; for T, n = 22 at Grade 2 and n = 20 at Grade 4. Random assignment of treatments to groups was made. Mean I.Q. for the two groups was slightly above average, about 110.

There were eight weeks of instruction, beginning in September, for 20 minutes per day, three days per week. Regular classroom teachers were trained and taught the classes. In grade 2, E subjects were taught these strategies for addition: doubles, sharing (compensation) to relate facts to doubles, adding to 9, counting on and using 10. For subtraction E subjects were prompted to think of the addition fact. In grade 4, E subjects were taught these multiplication strategies: patterns, relationships, and other techniques (e.g., twice as much, adding on, subtracting from, finger multiplication for 9s,

commutativity). For division, students were encouraged to think of the multiplication fact.

The instruction on facts for T-groups was based on the sequence in the adopted textbook with supplementary teacher-directed drills. Mathematics time for both T and E outside the fact-instruction time was on content topics such as geometry, time, or metric measurement.

The dependent measure was the number of facts correct on identical pretest, posttest, and retention test (two weeks) with 3-minute written timing. Data were analyzed using ANOVA with a general 2 x 3 design. Scheffe tests for pairwise comparisons and _t_ tests were carried out subsequently. Results for "harder facts" on addition in grade two and on multiplication in grade 2 were analyzed separately but in the same manner. The upper and lower thirds of each group were interviewed after the retention test in mid-November and again in May.

4. <u>Findings</u>

There was a marked effect in favor of E:

(a) In grade 2, $p < .001$ on the posttest and the retention test. on both addition and subtraction.

(b) In grade 4, $p < .001$ on the posttest in multiplication, $p < .05$ on division posttest and $p < .05$ on retention tests for both multiplication and division.

In grade 2, actual scores for T declined on subtraction from the pretest and gains in addition for T were small, about 7. In grade 4, E gained about 50 on multiplication while T gained about 30. On division, E gained 49 and T gained 27. Gains on harder facts for addition and multiplication were similar. E gained 11 on the posttest in grade 2 while T declined by 1. On multiplication, E gained 25 while T gained 9.

From the interview data it was discovered that about three-fourths of students in E used the strategies that were taught explicitly. About one-third of the students in T used strategies; the most evident strategy was the usual one of counting using a ruler or fingers. For facts mastered during the eight-week period, retention of strategies used earlier was more consistent for high and for low achievers.

5. Interpretations

The data support the use of thinking strategies to facilitate learning of basic facts. A discussion of the poor performance of T in grade 2 suggested that confidence of pupils in their ability to do the facts was a dominant influence. The author recommended that curriculum and classroom efforts focus more carefully on the development of thinking strategies prior to drill on basic facts.

Abstractor's Comments

The study holds substantial significance in mathematics education for several reasons:

(1) It reminds the mathematics education profession that older substantial studies need to be reexamined for replication and useful implications for problems evident in today's schools.

(2) It addresses one component of the popular return to the basics with the need to "know the facts."

(3) It provides evidence on the effectiveness of thinking and reasoning even on skill-oriented topics.

With a shift in the way facts are organized and taught, there appears to be a relatively easy way to improve achievement. Furthermore, while the evidence in this study is not as direct as one would like, the confidence of pupils who are taught the strategies seems to be strengthened.

The results found by the investigator on addition and multiplication seem quite dependable. The results for subtraction and division bear some skepticism because of the lack of specificity about the time the strategies were used and the duration of practice on these strategies. Furthermore, it was not clear just how the strategies for subtraction and multiplication were related to addition and multiplication, respectively.

The interview data on facts that were mastered suggest that such an experiment should extend over a longer period of time, perhaps a full school year, under a mastery-learning scheme.

Children seemed to follow the advice of skipping facts on which they were unsure. It needs to be determined if children do this as

easily on a pretest as on a posttest because there is some possibility that the substantial gains could have some spurious component. If children are more confident, they may be more willing to skip.

There were a few minor irritations in reading the research report. It took several readings to discover that the major dependent measure was the "number" of facts completed correctly in three minutes. In paragraph two, page 219, there is a statement on subtraction contrary to the data. The T_1, T_2, and T_3 used in Table 4 were not defined. It takes some study to realize that they probably refer to pretest, posttest, and retention test. In 1b, page 225, "counting on from the greater addend" was used as a general description of strategies for E even though the reference is to both addition and multiplication. An initial figure on the sequence of events lists a Transfer Period with Interim Tests yet there were no data and no discussion of this part of the study.

These minor inconsistencies should not detract from a very useful piece of work. It is the kind of study that is easy to replicate. With each replication there is likely to be good in-service for teachers as well. This is a happy combination indeed.

Vance, J. H. ATTITUDES TOWARD MATHEMATICS AND MATHEMATICS INSTRUCTION OF PROSPECTIVE ELEMENTARY TEACHERS. Alberta Journal of Educational Research 24: 164-172; September 1978.

Abstract and comments prepared for I.M.E. by THOMAS COONEY, University of Georgia.

1. Purpose

The purpose of the present study was to compare the attitudes toward mathematics and the beliefs about mathematics and mathematics instruction of prospective elementary teachers at three stages of preparation at the University of Victoria. The attitudes and beliefs of high and low achieving students, students enrolled in primary and intermediate sections of a methods course, and students enrolled in a methods course with and without a particular content course as a pre-requisite were also investigated.

2. Rationale

A basic assumption of the study is that the attitudes and beliefs of elementary school teachers toward mathematics affects the way in which mathematics is taught. Previous research indicates that pre-service elementary teachers tend to have more of an informal view of mathematics after their training in mathematics education than before the training.

3. Research Design and Procedures

Three Likert-type scales were used. The Attitude Toward Mathematics Scale (ATMS) measured interns' positive and negative feelings toward mathematics. The Belief About Mathematics Scale (BAMS) and The Belief About Mathematics Instruction Scale (BAMIS) were designed to measure a formal-informal dimension of belief. "A high score on the BAMS indicates an appreciation of the creative and investigative nature of mathematics, as opposed to a view that mathematics is a closed, rigid subject. A high score on the BAMIS suggests a strong belief in the importance of student exploration and discovery, as opposed to teacher explanation, in learning mathematics."

The scales were administered to various groups of prospective elementary teachers enrolled at the University of Victoria in the 1973-74 and 1974-75 academic years. Subjects were classified in two ways: by preparation and by achievement. The following three levels of preparation were defined:

i Beginning of math course for prospective elementary teachers

ii Completion of same course

iii Completion of mathematics methods course

Two levels of achievement were defined: high and low. The levels were based on high school or university mathematics courses completed and on grades received.

A 3 x 2 (preparation by achievement) design was used to compare scale scores of independent groups at the three stages and two achievement levels. Subjects per cell were 21, 28, 34, 42, 25, and 28. "At each stage, only those subjects who had responded to the scales for the first time at that stage were included in the sample. Some students were tested at two different stages. Data were obtained for 46 subjects who completed the scales prior to and upon completion of the mathematics course in 1974-1975 and for 27 subjects who completed the scales upon completion 1973-1974 and upon completion of the methods course in 1974-1975. Pre- and posttest scores were compared using the correlated t-test."

4. Findings

Students began the mathematics course with fairly positive attitudes toward mathematics. The attitudes of these students did not differ significantly from those who had completed the course or from those who had completed both the content and the methods course. Students who completed both the content and methods courses had a more informal view of mathematics than entering students or students who had completed just the content course. Students began the content course with neutral beliefs about mathematics instruction. Students developed a more informal view of instruction upon completion of the mathematics course and still more so after completion of the methods course. High-achieving students had more favorable attitudes and a more informal view of mathematics instruction than low achievers.

Analysis of data from students who completed the scales at two successive stages of preparation also resulted in several findings. Students who successfully completed the mathematics course tended to develop a more favorable attitude and a more informal view toward mathematics. Successful completion of the methods course tended to promote positive attitudes toward mathematics and informal beliefs about mathematics and mathematics instruction.

5. Interpretations

It appeared to the investigator that the mathematics education program was reasonably effective in developing desirable attitudes toward mathematics and mathematics instruction. High achievers tended to be more positive than low achievers. This is probably because teachers with a poor knowledge of mathematics are more likely to emphasize drill activities and feel more constrained in promoting students' exploration in mathematics. Transfer students and other students who had another mathematics course besides the one of interest here were found to have similar attitudes and beliefs upon completion of the methods course to those who had taken the standard course.

Abstractor's Comments

Some results seemed contradictory. One analysis indicated that attitudes of students at the three preparatory stages did not differ significantly. Yet it was reported that students who took the instruments on a "pre-post" basis developed a more favorable attitude. No explanation was given for the apparent conflict.

Still it is "comforting" to think that mathematics teacher education programs can have a positive effect on interns. However, the findings should be viewed in the context of other questions. To what extent do changes in attitudes and beliefs affect preactive or interactive phases of teaching? Do the changes in any way transcend beyond paper and pencil changes? If so, how? What specific aspects of a methods course are more likely to influence changes in attitudes and beliefs?

In fairness, these questions were not the concern of the present investigation. However, unless we know more about how attitudes and beliefs affect a teacher's performance and come to understand how changes in a teacher's effect occur, we have not truly gained much knowledge. Also, there is the additional concern that the instruments used in this and other similar studies provide only a rather narrow perspective on teacher effect and affect.

In closing, I found the article rather interesting and informative. But unless the basic problems of teacher education are dealt with in a more comprehensive manner, progress in upgrading our teacher education programs is not likely to be very fast coming.

Webb, Norman L. PROCESSES, CONCEPTUAL KNOWLEDGE, AND MATHEMATICAL PROBLEM-SOLVING ABILITY. Journal for Research in Mathematics Education 10: 83-93; March 1979.

Abstract and comments prepared for I.M.E. by LEN PIKAART, Ohio University.

1. Purpose

The study was designed to investigate the relationship and relative importance of (a) conceptual knowledge variables and (b) process variables in mathematical problem solving.

2. Rationale

Conceptual knowledge may be envisioned as the facts, concepts, principles, and algorithms that are needed to solve problems, whereas process refers to the techniques, strategies, or heuristics needed to recall and construct information while solving problems. Referring to an analysis by Mayer, Webb identifies the following three stages of a learning process: reception of material, existence of prerequisite knowledge (conceptual knowledge) and activation of assimilative set (process). He notes the difficulty of attempting to quantify the relative contribution to problem solving of conceptual knowledge and process because each construct is dependent on the other. That is, problem solving which is dependent on process in one instance may be part of learning and become conceptual knowledge in a later instance.

3. Research Design and Procedures

A sample of forty high school students taking second-year algebra was selected from a group of volunteers in four schools that represented different social and economic areas. Later, three subjects had to be eliminated.

A pretest, administered at each school, consisted of 16 scales of mathematics achievement, attitudes, mental abilities, and problem solving. All but one scale was selected from the National Longitudinal Study of Mathematical Abilities. Later, students were interviewed individually and tape recorded as they solved, thinking aloud, eight problems in a test called the "Problem-Solving Inventory." The following three sets of data were used to code the problem-solving protocols:

a) Heuristic strategies. Eighteen variables, such as "draws a representative diagram," were identified to represent problem-solving processes. (Originally "counting, algebraic, and other errors" were classified as checklist variables, but later these three variables were reclassified as process-sequence variables.) These variables were checked only once if the process was observed on each problem.

b) Process-sequence variables. Fifteen variables, such as "reads the problem" or "uses trial and error" were noted each time the student employed the process.

c) Score variables. Success in problem-solving was measured by a total score ranging from 0 to 5 on each problem with three subparts--approach (0-1), plan (0-2), and result (0-2).

Principle component analyses were performed on the 16 pretest scales, the 18 heuristic strategies scales, and the 15 process-sequence variables, reducing the number of variables to 4, 5, and 4, respectively. These constructs accounted for from 61% to 70% of the total variance in each set and were identified as follows:

Pretest Components	Heuristic Strategies Components	Process-Sequence Components
Mathematical Achievement	Pictorial Representation	Deductive Production
Verbal Reasoning	Product Checking	Random Production
Field-dependence	Concrete Representation	Non-production
Negative Anxiety	Recall	Recall Production
	Sudden Insight	

Finally, regression analyses were performed using the total score of the Problem-Solving Inventory as the dependent variable and changing the order of entering the independent variables above.

4. Findings and Interpretations

Tables of the regression analyses are presented depicting the results when Pretest Components were entered first and when Heuristic Strategies Components were entered first. However, when all components were introduced stepwise, the sequence of selection drew from both sets of components. Multiple R^2 reached 0.72 with 6 components, 3 Pretest and 3 Heuristic Strategies. "The large overlap of the components in accounting for vari-

ance in problem-solving scores provides evidence that conceptual knowledge and processes are interrelated."

Abstractor's Comments

The study is an important link in understanding the problem-solving process and should be of interest to all mathematics educators. It is well conceptualized, designed, and reported. Webb is well aware that it has limitations--subjects "...were not randomly selected to represent a particular population, ... the small number of problems prohibited making generalizations across a large number of problem-solving situations," the talking-aloud method may not have indicated all strategies being employed, the pretests were general measures of abilities rather than specific measures of conceptual knowledge needed to solve the problems, and the scoring technique for the dependent variable may have influenced the results.

It is important to note that the regression analyses lead to optimistic results. The multiple correlation is a statistic that mathematically optimizes the common variance between two sets of variables. That is, <u>at best</u> about 74% of the variance in the Problem-Solving Inventory can be accounted for by the other variables in the study.

Still, as an exploratory study, it presents evidence that both conceptual knowledge and process variables are important components of the ability to solve problems. It is to be hoped that other investigations will be conducted to explore the relationship in other samples of a population and to explore the effects of instructional treatments on the interrelation of these components.

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